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Progress, Problems, and Prospects in Meteorological Data Assimilation

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PROGRESS, PROBLEMS, AND PROSPECTS IN METEOROLOGICAL DATA ASSIMILATION

I. Introduction

The advent of satellite-borne radiometric sensors in the 1960's raised the possibility that nearly continuous observation of atmospheric temperature on a global basis would soon be possible for the first time. Aside from considerations of accuracy, this type of observation differs from conventional data in two fundamental ways. First, the satellite-derived data are incomplete, in that only one meteorological variable (temperature) is observed. Second, the observations are distributed in space and time, rather than at fixed locations and times. It became evident that, in order to make full use of the new data source, adjustments in the operational practice of numerical weather prediction would be necessary.

Research was therefore stimulated into methods by which the new asynoptic, incomplete data might best be utilized. Beginning with a paper by Charney, Halem, and Jastrow (1969), the intervening years have seen a considerable effort in what has come to be called four-dimensional data assimilation. Much progress has been made, both in understanding the nature and limitations of the data and in developing techniques for using it. However, many problems are as yet unresolved. This paper presents a review of the significant progress since the paper by Charney, et. al., and an assessment of the remaining problems. The final section speculates on the future of four-dimensional assimilation, the implementation of operational assimilation systems, and the likely impact on numerical weather prediction.

II. Some definitions

The literature in this area tends to be long on imaginative thought and experimentation, but short on discipline and perspective. This is reflected in the language: a peculiar jargon replete with terms amusingly liberated from the medical profession, which lose precision in the transfer. Moreover, there is no general consensus on what constitutes four-dimensional assimilation, and what does not. The following discussion is intended to establish some guidelines in terminology, which, if not universally accepted, will nevertheless be adhered to in this paper.

Assimilation implies a process by which something is absorbed into something else. In the present case, meteorological observations are absorbed into a numerical representation of the atmosphere. A common numerical representation is an "objective analysis," which is usually taken to mean an interpolation of irregularly-spaced observations to a regularly-spaced set of grid points. Assimilation may be distinguished from interpolation by requiring that the numerical representation into which the observations are absorbed be a complete description of meteorological variables whose mutual dependence is governed by explicit physical constraints: a set of model equations, for example. Thus, an assimilated state is one in which observations and numerical representation of the meteorological variables are interchangeable and obey prescribed physical constraints. In practice, the degree of interchangeability must depend upon the error levels of the observations and the constraints.

Such a state may be one-, two-, or three-, or four-dimensional. An example of two-dimensional assimilation results from Rutherford's (1973) procedure of simultaneous statistical weighting of mass and motion observations, subject to the constraint of geostrophicity. Flattery's (1971) representation of the atmosphere in terms of three-dimensional series of orthogonal functions may be considered a three-dimensional assimilation, for there is a dynamic constraint implied in the functions used to represent atmospheric variability. It is important for the extension of this argument to four dimensions to note that, in both of these examples, the numerical representation at any point in space is influenced by observations in all directions from the point in question. That is, the influence of each observation on the numerical representation is approximately isotropic. Similarly, in a four-dimensional representation, the influence of each observation must be approximately isotropic in time as well as in space. That is, the numerical representation at a given time should mutually "fit" all of the observations distributed through the interval.

Four-dimensional data assimilation, as subsequently referred to in this paper, will be taken to mean a process which results in a consistent representation of the complete meteorological variables over some definite time interval, reflecting a mutual accommodation to observations distributed in space and time and governed by a set of time-dependent constraints. This definition ascribes four-dimensionality to the result of a process, rather than to the process itself.

It thus represents a departure from the prevailing view evident in the literature. An assimilation is often referred to as four-dimensional merely if the process of arriving at an assimilation state involves time. To adopt this point of view renders difficult a distinction between four-dimensional assimilation and current practice at most operational numerical weather prediction centers. At present, most centers describe atmospheric evolution in terms of a sequence of three-dimensional representations at distinct intervals in time. Each is connected to its predecessors by a short-period extrapolation from the previous representation. Time is thus involved, and so this could be described as four-dimensional assimilation. But the remaining three-dimensional representations are not necessarily mutually consistent, and therefore do not satisfy the definition adopted here. Assimilation systems which perform in this way will be denoted as 3.5-dimensional in this paper.

Consider now the methodology of developing a four-dimensional assimilation system. Gandin (1968), Peterson (1968), Eddy (1974), and others suggest that it should be possible to extend the method used by Rutherford in two dimensions to the full four-dimensional problem. To distinguish this approach from others, it may be termed statistical assimilation. The method is, at present, of limited practicality, partly because of the difficulty of determining complicated statistical correlation functions, and partly because of the difficulty of incorporating non-linear, time-dependent constraints.

The latter difficulty also afflicts another method, which will be termed variational assimilation. Based on the calculus of variations, it is a mathematically elegant approach which has been pursued by Sasaki (1970), and others for a number of years. Lewis (1972) successfully developed a three-dimensional variational assimilation scheme, using a thermal wind relationship and the hydrostatic equation as constraints. However, the elegance of the method is overwhelmed by its difficulty of solution when general constraints are introduced.

Most of the research has therefore revolved around a less elegant but more practical method in which a numerical prediction model serves as an integrator of observations distributed in space and time. Morel and Talagrand (1974) refer to this as dynamic assimilation. In this approach, a continuing integration of the model is interrupted periodically and the current model representation is updated by timely observations. The act of updating the model state is referred to as inserting the data. Direct insertion refers to updating the model only at the model grid point nearest the location of an observation. Interpolating the observation to several nearby grid points prior to updating the model is termed indirect insertion. The temporal analogues of these two terms are continuous insertion, where updating is done at the model time step nearest the time of the observation, and intermittent insertion where the model is updated at distinct intervals by data stratified into groups.

At the instant of insertion, the observations and model representation are, by definition, interchangeable; but they do not necessarily remain so. For example, if observations of the mass field are inserted into a primitive equation model without a corresponding adjustment to the model representation of the motion field, an imbalance is created which, when the integration is resumed, is manifested as gravitational oscillations. This is referred to in the literature as the model shock. It may be interpreted as the model's attempt to restore the dynamic balance which would have existed in the absence of insertion. If, after a few time steps, the model is reversed and integrated backward to the insertion time (assuming that the model is reversible), it may return to a different model representation. If this final state is similar to that existing immediately prior to insertion, the impact of the inserted data is minimal: the model has forgotten the inserted data. This has been referred to in the literature as the rejection phenomenon. On the other hand, to the extent that the final state is similar to that existing immediately after insertion, the model has remembered the inserted data: the observations have been assimilated to some degree. If the model returns exactly to the state existing after insertion, model state and observations are interchangeable and obey the dynamic constraints of the model equations. Reinserting the observations would not disturb the model state; therefore, the existence of model shock is a symptom of imperfect assimilation.

In practice, the observations are imperfect, as are the dynamic constraints, and there will generally be more than one insertion during the interval for which a four-dimensional representation is desired. To obtain a consistent numerical representation, it is necessary to reconcile the inevitable conflicts between imperfect asynoptic observations and an imperfect model. Such reconciliation cannot be effected by dynamic assimilation alone, if the integration is always forward in time: any conflicts between observations at different times during the interval are resolved in favor of the latest data. The result may be a sequence of mutually inconsistent three-dimensional representations, and the assimilation system in this case would be described as 3.5-dimensional. If, however, the integration is cycled forward and backward over the interval containing the data, it is possible to reconcile conflicting observations on the basis of the dynamic constraints of the model and convergence to a fully four-dimensional representation may be possible. This system forms a basis for dynamic four-dimensional data assimilation.

III. A review of the literature

There exists a substantial body of literature on assimilation and related topics. Excellent reviews have been published by Kasahara (1972) and Jastrow and Halem (1973). All of the studies discussed in these two reviews dealt with simulation studies in which the "observations" to be assimilated were generated by a numerical prediction model: no experiments with real data had yet been performed. In this paper, emphasis is placed upon the few experiments in assimilating real data that have so far been published. Simulation studies are summarized mainly from the point of view of their contributions to the design and construction of real-data assimilation systems.

a. Simulation studies

The customary procedure for a simulation experiment begins with a "control" integration of a numerical prediction model for several days. The atmospheric representation thus produced is regarded as "truth," and "observations" are extracted from it. The assimilation integration begins from a different initial state, and periodically the "observations" are inserted. To the extent that the "observations" are assimilated, the model state progressively approaches the control state. Such a framework enables the experimenter to assess the impact of different configurations of observing systems, and different levels and distributions of observational errors against a known solution. There are also some serious disadvantages which will be discussed later in this section.

(1) The induction question

The first simulation study was due to Charney, Halem, and Jastrow (1969). Their object was to determine if the incomplete data provided by satellite sounding systems could be used to induce a complete representation of the meteorological variables: that is, can the motion field be determined from observations of the mass field? Various national weather services had been doing so, via diagnostic relationships such as the balance equation, in routine operations for several years. But such wind determinations were unsuited for low latitudes. Charney, et. al. proposed to determine the motion field for all latitudes with a more general mass-motion relationship: the primitive equations.

Using the simulation procedure described above, several experiments were conducted. Each consisted of a forward-only assimilation with direct insertion at all grid points intermittently at intervals ranging from 1 to 24 hours. The resulting numerical representations were therefore 3.5-dimensional in terms of the nomenclature introduced in the previous section. Insertion of the "correct" temperatures every 12 hours reduced the root-mean-square (RMS) wind error to less than 1 m sec^{-1}, but required approximately 20 days to reach an asymptotic level. The addition of random "errors" to the inserted data increased the asymptotic level of error in the assimilated winds. Larger asymptotic error levels resulted in the tropics than in middle latitudes.

These results indicated that the assimilation of temperature data into a numerical model can induce winds at all latitudes, although less accurately at low than at high latitudes. The question of whether the accuracy of the winds so obtained is superior to a solution of the balance equation in the extratropical region was not addressed. A comparison would have been interesting. Charney, et. al. carefully emphasized the significance of wind determination in the tropics, although the experiments were inconclusive with respect to the accuracy of the induced tropical winds. This properly drew attention to the tropics as the principal potential beneficiary of assimilation methods. Subsequent research was not always so careful to focus on the tropics. Indeed, as interest expanded, so did a tendency to view assimilation methods as inherently superior to established diagnostic methods, and to shift the emphasis away from the tropics to the higher latitudes.

A vigorous burst of activity followed the paper of Charney, et. al. examining the important factors in assimilation.

(2) Geostrophic adjustment

The theory of geostrophic adjustment (Monin and Obukhov, 1969; Washington, 1964) predicts that the motion field will adjust to the mass field for high latitudes and large scales of motion, and that the reverse is true for low latitudes and small scales. Williamson and Kasahara (1971) examined the extent to which the theory is valid for a sophisticated multilevel model. Using a large general circulation model, a series of 3.5-dimensional assimilation experiments were conducted. Following Charney, et. al., model-generated temperatures were inserted at intervals of 12 hours, and the accuracy of the induced winds examined as a function of latitude and of wavelength. It was found that the winds were more accurate at high latitudes and large scales, in agreement with theory. A reciprocal experiment was then performed, in which model-generated winds were inserted at 12-hour intervals. Temperatures were recovered with reasonable accuracy after 15-20 days. Higher accuracy was achieved for small scales but not for low latitudes, in partial agreement with theory.

In a later study, Williamson (1973) found that the error levels in the wind-induced temperatures were lowest in the intermediate rather than the smallest scales, and in mid-latitudes rather than the tropics. He speculated that the scale disagreement was produced by large truncation errors in the model in very small scales. It seems possible that modeling difficulties, such as inaccurate physical approximations, might contribute to the latitudinal discrepancy.

Rutherford and Asselin (1972) also examined the latitude and scale dependence of assimilated winds in a series of experiments with a primitive-equation barotropic model. Model-generated 500-mb height observations were inserted periodically over a 10-day period. They concluded that the accuracy of the induced winds was acceptable only for the largest scales in middle and high latitudes, and that wind observations would be necessary to delineate the small scales.

(3) Tropical wind experiments

Further insight into the adjustment process in the tropics and elsewhere was obtained from a series of Observing Systems Simulation Experiments conducted to investigate alternative configurations of existing and proposed observation systems. The expressed aim was to determine whether wind observations would be necessary in the tropics, or whether acceptably accurate winds could be induced from mass observations alone. The results have been reported by Gordon, et. al. (1972), Kasahara and Williamson (1972) and Jastrow and Halem (1973).

Each group used its own general circulation model as the main element in a 3.5-dimensional assimilation experiment. Simulated temperatures and surface pressures were inserted at 12-hour intervals for periods of 20 days or more in each experiment. An augmentation corresponding to the availability of cloud-tracked wind vectors was simulated by inserting model-generated winds at one low level and one high level in the tropical belt between approximately 25N to 25S. Finally, winds were inserted at all levels in a narrow equatorial zone (∿ 10N-10S).

The results indicated that insertion of temperatures and surface pressures are insufficient to induce tropical winds to the accuracy specified by the Joint Organizing Committee of the Global Atmospheric Research Program (GARP). Wind observations at two levels in the tropical zone were of assistance, but also insufficient. The addition of detailed equatorial winds, however, did reduce wind errors to acceptable levels. Gordon, et. al. also found that the temperatures in the tropics responded favorably to the full wind specification, in agreement with adjustment theory. The discrepancy between this result and the earlier result of Williamson and Kasahara has not been explained, but it is noteworthy that there were differences in vertical resolution and modeling approximations in the two models.

(4) Reference level experiments

Another important question addressed by various simulation studies is whether it it necessary to specify the meteorological variables at some geopotential altitude to serve as a reference level for hydrostatic integrations. Kasahara (1972) presented a thorough discussion of the reference level question; a repetition is not necessary here. In summary, experiments in which temperatures were inserted with and without surface pressures or, for example, geometric height of some constant pressure surface, indicate that a reference level is needed. Jastrow and Halem (1973), surveying several such experiments, concluded that accurate specification of surface pressure should be an important objective of any proposed global observing system.

(5) Continuous vs. intermittent insertion

Charney, et. al. inserted artificial observations at every grid point at distinct intervals during the assimilation integration. With the possibility of nearly continuous observations from polar-orbiting satellites, the question arose as to whether any advantage might accrue from inserting the observations continuously at the nearest model time to the real time of observation rather than intermittently in batches Jastrow and Halem (1970) reported on an experiment in which the data were inserted following the path of the sounding satellite. The results did not differ in any fundamental way from those of Charney, et. al., a fact later confirmed by Jastrow and Halem (1973). Independently, Morel and Talagrand (1974) concluded that there is very little difference with simulated data introduced at 12-hour intervals and the same amount of data distributed over a 12-hour period.

(6) Insertion frequency and data quantity

Most investigators therefore used intermittent insertion, and many experiments were conducted to determine an optimum frequency of insertion. Charney, et. al. inserted at every grid point at 1-, 6-, 12-, and 24-hour intervals and found that 12-hour insertion resulted in the greatest error reduction. Williamson and Kasahara (1971) experimented with 2-, 4-, 6-, and 12-hour insertion intervals, also inserting at every grid point, and concluded that the 4-hour interval was superior. It may be noted that, in both of these experiments, an increase in the insertion frequency was accompanied by an increase in the total amount of data inserted per model day and a decrease in the time available to dissipate the model shock between insertions. Morel, Lefevre, and Rabreau (1971) argued that there is an optimum amount of data beyond which redundancies will contribute to gravity waves. Talagrand (1972) showed that the optimum insertion frequency depends on the damping characteristics of the prediction model; more rapid suppression of noise is associated with higher permissible insertion frequencies. Morel and Talagrand extended this argument to relate the insertion frequency to the damping rate of gravity wave noise, and to the predictability error growth rate of the model. They contend that data should be inserted sufficiently frequently that the model does not "forget" between insertions, and sufficiently infrequently that gravitational oscillation resulting from one insertion are dissipated before the next. The insertion frequency is thus model-dependent to some degree.

(7) Damping methods

Therefore, an important element in a dynamic assimilation system is a technique for dissipating the model shock quickly and efficiently. Various techniques have been used by different authors. Talagrand, for example, used the Euler-backward (Kurihara, 1965) time integration method, which damps at a rate proportional to the frequency of oscillation. Rutherford and Asselin (1972) performed assimilation experiments with a semi-implicit time integration method which is neutral in itself (Kwizak and Robert, 1971), but used a time filter (Robert, 1966; Asselin, 1972) to suppress the model shock. Morel and Talagrand (1974) support a special viscosity term which affects only the divergent component of the wind, contending that this device is much more selective than is the Euler-backward integration method. Mesinger (1972) reported that a comparison between the Euler-backward method and a generalization of the Heun-Matsuno method favored the latter, which possesses superior damping capability. No extensive direct comparison of the various methods has been carried out, although Halberstam (1974) has recently published a comparison between a damping method similar to the Euler-backward and two methods which are essentially neutral. He found that the damping method was superior. The sum of the evidence strongly suggests that some form of damping is necessary, but support for any particular method is not yet convincing.

(8) Data quality and indirect insertion

A unique characteristic of simulation studies is that, with error-free, self-generated initial data, a prediction model will make an error-free forecast with respect to its own history (Williamson, 1973). While interesting, it is a poor approximation to reality, and most experimenters have added errors of various distributions and magnitudes to the simulated observations. The first experiments of Charney, et. al. were done with "perfect" data, but were repeated with random temperature errors of as much as 1C. With contaminated observations, the asymptotic level of wind error reached after 20 days was generally higher and proportional to the error level in the observations. Williamson and Kasahara (1971), again imposing random errors on model-generated data, found a relationship between wind and temperature errors.

Random observational error is not the only source of trouble with real data. A particularly worrisome problem with respect to some satellite sounding systems is their inability to penetrate cloudiness, thus suggesting large systematic errors in the vicinity of vigorous cyclones. Also, the procedures by which observed radiances are transferred to temperature measurements (Wark and Fleming, 1966) may lead to the observation errors being spatially correlated (Bengtsson and Gustavsson, 1972). Jastrow and Halem (1970) performed a series of experiments simulating incomplete coverage of satellite-derived observations due to cloudiness. Not surprisingly, they found an increase in the asymptotic error level of the induced winds. Later, the tropical wind experiments reported by Kasahara and Williamson (1972) and Gordon, et. al. (1972) were conducted with both random and systematic errors added to the model-generated data.

These simulation experiments were intended primarily to give some estimate of the impact of observational errors on variables induced by assimilation. At least as important is the problem of minimizing observational errors. In the simulation experiments, the "data" are generated at grid points and inserted at grid points; there is no necessity of interpolation. With real data, such convenience is not available, and some form of spatial interpolation is necessary. Miyakoda and Talagrand (1971) recognized the utility of the optimum interpolation procedure (Gandin, 1963) to reduce the impact of observational errors in data assimilation. In a continuation of that study, Talagrand and Miyakoda (1971) attempted direct insertion of error-contaminated, simulated data into a balanced barotropic model, only to encounter numerical instability. The cause of the instability is not clear, but indirect insertion via optimum interpolation did not cause the model to become unstable. Independently, Bengtsson and Gustavsson (1971) also used optimum interpolation of simulated data and concluded that its use accelerated the assimilation process. These experiments strongly suggest that a spatial interpolation system should be an important element of a data assimilation system.

It may be noted that the preponderance of these papers on insertion techniques dealt with the extratropics; virtually no attention was given to insertion in the tropics, the original primary focus of assimilation methods.

(9) An iterative method for four-dimensional assimilation

All of the studies discussed thus far, save two, were 3.5-dimensional assimilations, according to the terminology adopted for this review. The exceptions, papers by Morel, Lefevre, and Rabreau (1971), and later, Morel and Talagrand (1974) deserve special attention, for the approach they suggested is, in principle, the sole fully four-dimensional dynamic assimilation scheme yet to appear in the literature.

In most of the 3.5-dimensional simulation experiments, more-or-less "correct" data were inserted over a period of many days into an integration which started from a "poor" initial state. The assimilation process was assumed complete when the errors in an induced variable (e.g., wind, if temperatures are inserted) reached a small asymptotic value. No attempt was made to ensure that any particular inserted data set was also assimilated. Morel, et. al. (1971) proposed an iterative procedure in which a numerical model is integrated forward and backward over an interval containing the data, repeatedly inserting each data set until it becomes absorbed into the numerical representation. This process allows mutual adjustment between individual data sets. The resulting representation at any time within the interval is a mutual accommodation of all the data in the interval; i.e., it is fully four-dimensional. With simulated data and a primitive-equation barotropic model, their experiments showed that cyclical integration with repeated insertion over a definite period will converge to an optimum representation reflecting the data distributed over the period and subject to the dynamic constraints of the model equations.

With some reservations, this configuration is well suited to the requirements of operational numerical analysis and prediction, in which the available data is distributed over a 12-hour period and the problem is to assimilate that data into an optimum initial state for numerical forecasts. However, the experiments of Morel, et. al. showed the convergence to an assimilated state to be slow, equivalent to 10-15 days of 3.5-dimensional assimilation. For a multilevel, high-resolution model, the computational requirements would make this method too expensive for practical use, unless methods can be found to accelerate the convergence. This topic will be addressed further in later sections of this paper.

b. The transition to real-data experiments

From an early stage, it was recognized that knowing the "truth," in a simulation experiment, does not set one free: it is both the major strength and the major weakness of the method. Jastrow and Halem (1973) have presented a concise summary of the weaknesses they refer to as "model-dependence" and "incompatability." Both arise from the same source: the fact that the data to be inserted are generated by the same model that they are intended to correct. This means that both data and model share the same physical and numerical heritage, and are therefore more similar to each other than either is to the real atmosphere. When such observations are inserted, the model naturally finds them compatible, so that the shock of insertion is minimized. Real data, by comparison, are much less compatible with any existing model. The results of simulation experiments may therefore be qualitatively reasonable, but quantitatively overoptimistic.

Williamson (1973) has examined the effect of numerical errors in simulation studies. In his case, "truth" was an integration of a high-resolution version of a general circulation model, and the "data" extracted from this integration were inserted into a model of identical physical characteristics but of half the resolution. He found that the asymptotic error levels after 15-20 days of integration were much higher than in experiments where the data were extracted from the coarse-resolution integration. A similar result was reported by Morel, et. al. (1971) when data generated by a multilevel general circulation model were inserted into a primitive equation barotropic model.

Aware of these problems, Bengtsson and Gustavsson (1972) published a series of three experiments beginning with a pure simulation and progressing through assimilation of model-independent data to the ultimate challenge of real observations. In the first experiment, data generated by a balanced barotropic model were used to simulate both the conventional observing network as well as soundings from an orbiting earth satellite. Assimilating these data independently, they concluded that the observations derived from the satellite were more useful in providing accurate atmospheric representations. However, in the second experiment, the data were generated by the temporal interpolation of (independent) operational analyses. In this case, the errors in the assimilated state were much larger, and the benefit of the satellite was not so apparent. The third experiment assimilated 142 real observations from the NIMBUS IV satellite. No conventional data were used. The resulting 500-mb analyses, while deficient in some reports, were nonetheless quite respectable. This is the earliest published account of an assimilation of real data, and therefore is a noteworthy contribution.

Of more lasting significance, Bengtsson and Gustavsson pointed out some of the elements likely to be of importance for an operational assimilation system:

(1) an accurate short-range prediction model;

(2) interpolation of observations to model grid points to reduce the impact of observational errors;

(3) use of climatology as a background to ensure boundedness of assimilations;

(4) local balancing of the motion field to observations of the mass field to reduce the shock of insertion.

All of these elements appear in some form in the experiments with real-data assimilation, discussed in the following section.

c. <u>Real-data experiments</u>

Hayden (1973) published the first paper to deal exclusively with the assimilation of real data. The experiments were conducted with a two-layer primitive-equation model integrated on a polar stereographic projection of the Northern Hemisphere. Thus, the tropics were effectively excluded from his experiments. Into this model, temperature observations calculated from NIMBUS IV soundings were inserted at intervals of 2 hours. Data from the conventional observing network were excluded, except for surface pressure in a few of the experiments.

The characteristics of the model deserve some attention, recalling the statement of Bengtsson and Gustavsson (1972) on the necessity of using an accurate short-range model in assimilation. Resolution in the horizontal was 381 km at 60N, which is respectable; however, vertical resolution was very poor with only two layers. Pressure was used as the vertical coordinate; the effects of orography were excluded, as were parameterizations of various physical processes. Spatial differencing was accomplished with the semi-momentum method (Shuman and Vanderman, 1966), and the "leapfrog" method was used for temporal differencing. No explicit viscosity or frequency-selective damping methods were used, except as noted below. Hayden found the model to be reasonably well-behaved numerically, although its deficiencies, notably the poor vertical resolution, preclude it from being an accurate model. Nevertheless, this does not detract from Hayden's paper, for his purpose was primarily to test insertion techniques and to demonstrate feasibility, rather than to produce competitively accurate numerical representations of the atmosphere.

Hayden's choice of insertion techniques was heavily influenced by the experience of the simulation studies. At each insertion time, the temperature observations from a satellite orbit were interpolated to nearby grid points by means of distance-dependent weighting functions (Bergthörssen and Döös, 1955), using the current model state as a "first-guess." This has the effect of adjusting the model representation of the mass field to reflect the observations. From the gradients of the adjusted mass field, geostrophic corrections to the motion field were computed. This step represents the local balancing suggested by Bengtsson and Gustavsson (1972). After adjusting both fields, dynamic initialization, as suggested by Nitta and Hovermale (1969), was used to suppress the residual shock of insertion. Finally, Hayden used the cycling approach of Morel, et. al. (1971). One to three complete cycles, each consisting of 12 hours' forward integration followed by 12 hours of backward integration, were used in each experiment. The resulting assimilated states may therefore be regarded as four-dimensional.

In the simulation studies, evaluation of each experiment was straightforward in principle, since the truth could be regarded as known. With real data, this is not the case, and consideration must be given to criteria for successful assimilation. Hayden used three measures. First, the RMS surface vertical pressure velocity ($\omega = dp/dt$) is used as an indicator of noise produced by data insertion. It has been noted previously that the presence of noise is indicative of imperfect assimilation. Second, a measure of the model's memory was devised. Prior to each insertion, the RMS difference, ε_f, between the observations and the model representation interpolated to the observation locations was computed. If the model retains the impact of the inserted data, this difference should decrease with each cycle. Third, an estimate of the accuracy of the resulting model representation was obtained by forming RMS differences and correlation coefficients between departures of the assimilated state from the initial state and of an operational verifying analysis from the initial state. The verifying analysis included a complete data base of both conventional and satellite observations. Presumably, the assimilated data act to adjust the initial state toward the verifying analysis. If this is the case, the RMS difference should decrease and the correlation coefficient should increase with each cycle.

Hayden conducted experiments from three initial conditions: one terrible, one moderately terrible, and, finally, the best available: the operational verifying analysis.

The results indicated that improving the initial state accelerates the assimilation process. In an operational framework, the initial state would be the result of an assimilation over the preceding period and would therefore be quite a good representation of the atmosphere; assimilation of new data would presumably result in small changes. With Hayden's most accurate initial state, one full cycle (two insertions of each orbit), together with the geostrophic wind correction, sufficed to produce an assimilated state with a low noise level. The geostrophic wind correction significantly enhanced the model's memory, but dynamic initialization through iterating around each insertion proved to be unnecessary.

Hayden's experiments demonstrate that real observations derived from satellite radiance measurements can be assimilated in an operationally feasible configuration, by using insertion techniques suggested from the simulation experiments. Kistler and McPherson (1975) reexamined one of those techniques--the geostrophic wind correction--within the framework of a simpler, primitive-equation barotropic model. 500-mb height data calculated from satellite soundings were inserted into the model via a successive-approximation interpolation method (Cressman, 1959) as the model was cycled forward and backward over a 12-hour interval containing the data.

The experiments were evaluated by a refinement of Hayden's second criterion, dealing with the model memory. Immediately prior to each insertion, the RMS difference ε_f between the observations and the current model state was computed, as Hayden did. Following each insertion, a similar RMS difference ε_a was also computed.

The latter quantity represents the "fit" of the model state to the data at the time of insertion. If the data are corrective, $\varepsilon_a < \varepsilon_f$. When the model reaches the same insertion time on the next half-cycle, ε_f is recomputed, and the difference between ε_f^k (on the (k)th half-cycle) and ε_a^{k-1} is regarded as a measure of the model's "memory" of the inserted data. If $\varepsilon_f^k = \varepsilon_a^{k-1}$, the observations have been completely retained; if $\varepsilon_f^k = \varepsilon_f^{k-1}$, the observations have been completely rejected. Thus, for successful assimilation, one expects the difference between ε_a and ε_f on succeeding insertions to decrease. However, this is an insufficient criterion by itself, since it does not depend on the quality of the data inserted, only that what is inserted is remembered. It is necessary also to require that ε_a decrease during the assimilation process to a level near the expected error level of the data.

Experiments with and without the geostrophic wind correction confirmed Hayden's finding that adjustment of the motion field from mass field observations is necessary. With the motion adjustment, the observations were successfully assimilated after two insertions; without it, 20 insertions were insufficient to bring about an assimilated state. However, the geostrophic wind correction may not be used in low latitudes. Neither Hayden nor Kistler and McPherson examined suitable substitutes for the tropics.

Gauntlett and Seaman (1974) performed a series of experiments generally similar to those of Hayden, but differing in important details. A six-level primitive equation model with a σ-vertical coordinate was used in the experiments. Various physical formulations, including precipitation, orography, and convection, were incorporated into the model. It is reasonable to assume that this model is more accurate in short-range prediction than is the model used by Hayden. Their insertion procedure consisted of interpolating satellite-derived temperatures to model grid points using a successive-correction method. The current model representation in the σ-vertical coordinate was used as a first guess. Once the horizontal interpolation was performed, new wind components were calculated by either the geostrophic or gradient wind relationships. This is a very important departure from Hayden's procedure. Hayden replaced the model geostrophic wind component with that calculated from the adjusted mass field, leaving the ageostrophic model component unchanged. The procedure of Gauntlett and Seaman yields not an adjustment to the model winds, but completely diagnostically determined winds at each insertion. In order to overcome the resulting imbalance, dynamic initialization following Nitta and Hovermale (1969) was used for 90 time steps immediately after insertion.

The assimilation integration was forward only in time, and extended over a 48-hour period during which satellite-derived termperatures and/or surface pressure were inserted at intervals of 2, 6, or 12 hours. Two main questions were addressed: Is reference-level information necessary, and is there an optimum insertion frequency? The experiments were evaluated subjectively as well as by two objective measures. Error statistics of the assimilated states compared to conventional data withheld from the assimilation were computed, and forecasts were made from the assimilated states and verified against analyses. Both of these measures focus not on how well the data were assimilated, but on whether the data made any difference in the resulting initial states and predictions.

The results indicated that insertion of temperatures alone produced inferior assimilated states and 24-hour forecasts than insertion of temperatures together with surface pressures. Comparison of a 48-hour assimilation integration against a prediction over the same period, but without insertion, showed that the assimilation produced much better agreement with verifying analyses, thus indicating the value of the temperature and surface pressure data. Their experiments with regard to insertion frequency were not as informative, but some advantage with 2-hour insertion was noted.

Only observations of the mass field were inserted in each of the three papers just discussed. Corresponding adjustments to the motion field were made, in one form or another, but no wind observations were used. In reality, the observing network already contains conventional rawinsonde reports, aircraft winds, and estimates derived from tracking cloud elements. This data base will no doubt increase in the next decade as a network of geosynchronous satellites is established, perhaps augmented by other wind observing systems. The problem confronting designers of data assimilation systems is therefore to blend all of the available information ... forecasts, climatology, and data from disparate observing systems ... into the best possible four-dimensional representation of the atmosphere.

An important step in this direction has been taken by Rutherford (1973), whose work was referred to in the introduction as an example of two-dimensional assimilation. Such a description refers only to the specific application Rutherford described--an assimilation of 500-mb height and wind observations--and does not do justice to the potential of the approach. In principle, it can be extended to three space dimensions, and, in conjunction with an accurate short-range prediction model, has considerable promise as a viable 3.5- or 4-dimensional assimilation system capable of handling a heterogeneous data base.

The main element of Rutherford's method is a multivariate optimum interpolation procedure in which both mass and motion observations are simultaneously treated, subject to a weak constraint of geostrophicity.

The quantities interpolated are deviations of the observations from a current prediction; the statistical weighting functions used in the interpolation procedure are calculated under the assumption that the deviations are related through the geostrophic equation. This has the effect of adjusting the geostrophic component of the wind toward that implied by observations of the mass field, which is similar to but not as strong as the one-for-one replacement used by Hayden. The mass field is also adjusted to reflect the wind observations. Moreover, the weighting functions depend on the error characteristics of the observation; thus, data from different sources ... including forecasts and climatology ... may be allotted more or less influence, depending on their relative accuracy.

Rutherford used this method in assimilation experiments in conjunction with a primitive-equation barotropic model integrated on a polar stereographic projection of the Northern Hemisphere. A semi-implicit time integration scheme (Kwizak and Robert, 1971) was used with a time filter (Asselin, 1972) to suppress gravity wave noise. Conventional 500-mb height and wind observations were inserted via the multivariate interpolation scheme every 12 hours during a forward integration of 10 days. Each experiment was evaluated by calculating ε_a and ε_f for each insertion, for both heights and winds, and then averaging the results over all 20 insertions.

A total of eight experiments were made in which various combinations of data and constraints were employed. Best results, in terms of lowest ε_a and ε_f, were achieved when a univariate interpolation of heights was followed by a balance-equation solution for the wind first-guess, and then a univariate interpolation of winds. Height observations thus influenced the wind interpolation, but the reverse influence was not permitted. However, the result of a multivariate analysis of both height and winds was a close second. As Rutherford noted, the latter should be preferred for operational use in view of the expense of solving a balance equation for a large model.

d. Summary

From this considerable body of literature, there emerge a few general propositions that seem reasonably well established by the weight of accumulated experience.

It is clear that winds can be determined from observations of the mass field, at least in the extratropics, through assimilation. The accuracy of such a determination is probably superior to a purely diagnostic determination, but may still be insufficient. Observations of the motion field appear to be necessary, especially in the tropics.

A reference level also appears to be necessary for successful assimilation of satellite-derived observations of the mass field. The preferred location is the earth's surface.

There is little advantage, and perhaps some harm, in continuous insertion. Intermittent adjustment of the model is preferable. However, the optimum frequency of insertion, if such exists, has a very compliciated dependence on model characteristics, insertion techniques, accuracy of the inserted data, and data quantity.

Finally, an outline may be discerned of the elements of an assimilation system suitable for operational use. Foremost is a high-resolution prediction model capable of accurate short-range forecasts. An indirect insertion procedure, with interpolation of observations to model grid points, is necessary to reduce the damaging influence of observational errors and the resulting model shock. Provision must be made for damping residual gravitational noise. The evidence strongly suggests that a method for making adjustments to the motion field corresponding to observed adjustments in the mass field is essential for successful assimilation.

Many of the details, and a few other general questions, are not so well established and remain as problems to be solved.

IV. Some outstanding problems

a. The tropics

The foremost unresolved question, and still the principal task of data assimilation, remains unchanged from the first experiments of Charney, et. al.: how to proceed in the tropics. Simulation studies have illuminated some aspects of assimilation in the tropics. The evidence indicates that the mass and motion fields in low latitudes are essentially decoupled; therefore observations of both are necessary for a complete specification. Simultaneous insertion of independent mass and motion observations has not been attempted in any simulation, nor has the response of the model to such insertion been examined. None of the real-data experiments have dealt with the tropics. Rutherford has addressed the problem of simultaneous mass-motion insertion, but the constraint he used is not appropriate for low latitudes. The reaction of the model to insertion is likely to be far more important in the conditionally-unstable tropics.

Moreover, no attention whatsoever has been given to the problem of moisture observations. Smagorinsky, et. al. (1969) suggest that humidity measurements are redundant; a carefully constructed model will generate respectable moisture patterns from climatology, given a few hours. It may be that this conclusion should be reexamined within the context of data assimilation.

At this writing, therefore, the state of knowledge with respect to assimilation in the tropics is entirely unsatisfactory. The author is aware of real-data experiments currently being conducted by several research groups with data collected during the GARP Atlantic Tropical Experiment of 1974. These experiments will no doubt advance understanding of low-latitude assimilation.

b. Heterogeneous data base

Related to the problem of the tropics, but applicable outside the tropics as well, is the appropriate treatment of heterogeneous data from observing systems of different characteristics. The bulk of the observations which assimilation methods were conceived to treat will be temperatures obtained from radiance measurements on board polar-orbiting earth satellites and motion estimates derived from the tracking of cloud elements by visual and infrared cloud imagery. The former is not similar to a radiosonde, nor is the latter similar to tracking a surface-launched balloon. Satellite-derived temperatures are most correctly interpreted as means over relatively deep atmospheric layers and over some moderately broad horizontal field of view. Profiles of these temperatures necessarily display less resolution than is apparent in radiosonde profiles. This does not necessarily imply that radiance-derived temperatures are inferior, but merely that there is a difference. An assimilation system must be capable of discriminating between observations from different systems and extract maximum information from each. Similarly, the cloud-tracked wind estimates are available generally at low and high altitudes only, with considerably uncertainty in the exact altitude of applicability. The estimates also contain error due to mixed motions affecting the apparent movement of the cloud element. Nevertheless, these data contain information that may be used to advantage. The problem is to use it intelligently.

c. Engineering problems

Although the outline of a workable assimilation system has emerged from experimentation, a host of specific problems associated with its design and construction remain. Bengtsson and Gustavsson (1972) have noted the requirements for a prediction model capable of accurate short-range prediction on all scales. The degree of sophistication of physical modeling necessary for adequate accuracy needs to be examined. Morel and Talagrand (1974) state " ... energy sources and sinks active in the real atmosphere do not play a significant dynamic role during the relatively short periods of model time (12 to 24 hours) needed for restoring the geostrophic balance." This argument implies that the parameterization of certain physical processes ... for example, radiation and convection ... could perhaps be deleted in an assimilation system without adverse effect. On the other hand, convection is generally recognized as the primary driving force in the tropics. Failure to include its effects in the assimilation system could be detrimental.

The model's accuracy is also a function of resolution. Ability to resolve the significant scales of motion in the horizontal depends in part on the spacing between grid points (or the number of wave components in a spectral model), as does the phase error of the subsequent forecast. For very short periods (3-6 hours), the cumulative effect of truncation error on predicted phases is not likely to be excessive. Furthermore, recent advances in spectral methods make it likely that future assimilation systems may be based on spectral models, in which case truncation error may be further reduced. Nevertheless, the question of what constitutes adequate resolution is of great practical significance in designing an economically feasible assimilation system.

In the vertical, there is also a question of resolution. At present, most interpolation procedures employed to transfer observations to grid points result in a three-dimensional representation of the atmosphere which is, in reality, a "stack" of two-dimensional interpolations on constant-pressure surfaces, perhaps loosely related hydrostatically. For use in models in which the vertical coordinate is normalized pressure (σ-coordinate), a further interpolation in the vertical is necessary. Gauntlett and Seaman (1974) have commented on the errors introduced by this procedure, and on the desirability of fully three-dimensional interpolation. This will no doubt prove beneficial, but regions of sharp gradients such as near the earth's surface and the tropopause require additional resolution for adequate representation.

A three-dimensional extension of the multivariate interpolation procedure of Rutherford requires some knowledge, or at least intelligent assumptions, concerning the form of the correlation functions used in the procedure. Some two-dimensional calculations have been performed (see, for example, Gandin, 1963; Alaka, et. al., 1972; Schlatter, 1975), but only from radiosonde observations. Similar computations for satellite-derived observations are exceedingly difficult because such observations are not fixed in space or time. Moreover, little is known about the latitudinal variability of the correlation functions. The sensitivity of the resulting interpolations to such variability needs to be determined.

Schlatter, Rutherford, and Bengtsson and Gustavsson chose to interpolate deviations of the observations from a prediction valid at the time and place of each observation. Bengtsson and Gustavsson also noted the desirability of including climatological "observations" as a means of restraining the model in areas which are observed infrequently. One could as easily interpolate deviations from climatology and treat the current prediction at each grid point as an "observation." There are advantages and disadvantages to both procedures. In any case, observations, predictions, and climatology all contain information to be assimilation into the best possible representation of the atmosphere.

Another problem yet to be resolved concerns the optimum procedure for damping the gravity wave noise resulting from insertion in a multi-level model. Reference has already been made to several studies of the effectiveness of various procedures, but all were based on one-level models which have only one mode of response to mass-motion imbalances. By contrast, a model with N levels in the vertical has N free modes ranging from the high-frequency "external" gravity wave to very low frequency internal modes. Morel and Talagrand (1974) contend that the divergence-damping viscosity term should be superior to frequency-dependent damping schemes in a multilevel model because of its greater selectivity. However, since precipitation is intimately related to divergence, it is likely that the viscosity term will tend to adversely affect moist physical processes. The degree to which this is the case, and its significance with respect to the resulting assimilated states, needs to be assessed as a part of a general investigation of the response of multilevel models to insertion of data.

Part of that response is a purely numerical problem which arises when the time integration of the model is performed by means of the "leapfrog" method or, for that matter, almost any method involving three time levels. As Halberstam (1974) and others have noted, insertion at any one time will inevitably lead to a separation of solutions between even- and odd-numbered time steps in the absence of any controls on such behavior. Predictor-corrector integration methods, such as the Euler-backward, do not permit this behavior but are generally too expensive to use throughout an assimilation integration. A common practice in many assimilation experiments has been to "restart" the integration following each insertion with a predictor-corrector method but then revert to the more economical leapfrog method after one or two time steps. Sundström (private communication, 1974) has noted that this introduces a truncation error, even where data have not been inserted, and suggests that this error, and the separation of solutions, can be avoided by adjusting two adjacent time levels at insertion. This promises to be a relatively inexpensive remedy.

The details of combining this procedure with some form of local balancing are not yet clear. Indeed, the optimum form of the balancing procedure itself has not yet been determined. Rutherford's (1973) results suggest slight superiority of balancing procedures more general than the geostrophic constraint, but that superiority must be balanced against the greater expense. In addition to the form of the procedure, the degree to which its mass-motion relationship should be enforced also required examination. Direct replacement of predicted geostrophic wind components by those computed from observations (Hayden, 1973; Kistler and McPherson, 1974) leads to an acceleration of the assimilation process through reinforcement of the model's memory. But it is likely that occasions may arise in which the quality of the inserted data is suspect; in such cases, good memory of bad data is not a virtue. The weaker constraint as used by Rutherford has much more appeal. Rutherford's approach also has the advantage of allowing the constraint to be enforced to a latitudinally-dependent extent, thus permitting greater independence between mass and motion fields in low latitudes.

Mass and motion fields so decoupled may constitute a complete representation of the tropical atmosphere, but it does not follow that the fields represent an assimilation as defined in the introduction. There are the additional requirements of mutual consistency in space and time and obedience to time-dependent physical constraints to be satisfied. How these requirements are to be imposed is as yet undetermined.

V. Speculations on the future of dynamic four-dimensional assimilation

A distinction has been drawn in this paper between dynamic assimilation experiments in which the integration is always forward in time and those in which the model is cycled both forward and backward over an interval containing the data. The former have been denoted as 3.5-dimensional, the latter as fully four-dimensional, assimilations. It is reasonable to enquire as to whether this distinction is real and, if so, whether it is significant.

A 3.5-dimensional assimilation results in a sequence of spatial "snapshots" of the atmosphere. Each may reflect the observations valid for one time, in mutual accommodation and obeying some set of constraints. Time enters only as a short-period extrapolation from the previous snapshot. This differs very little from the established practice of the U. S. National Meteorological Center, and most other operational centers, over the two decades of routine numerical weather prediction. In 3.5-dimensional assimilation, the observations are inserted by means of a "local" interpolation ... i.e., affecting only the grid points in the vicinity of the observations .., and at more frequent intervals corresponding to an orbit or two of satellite-derived data.

In operational practice, the data are stratified into larger time blocks (e.g., 12 hours), and are inserted by a general interpolation, or "objective analysis," presumably affecting all grid points. The difference between operational practice and 3.5-dimensional assimilation becomes thin when it is recognized that the operational interpolation procedure merely modifies a current prediction to reflect available data; where there are no data, the interpolation returns the prediction virtually unmodified. It will become thinner still as more operational centers reduce their cycle periods from 12 to 6 hours, as the Canadian Meteorological Centre and others have done.

On the other hand, four-dimensional dynamic assimilation in principle results in a virtually continuous representation of atmospheric behavior over an interval of time, e.g., 12 hours. At any particular time during the interval, the spatial snapshot is one which reflects the data not only at that time, but all of the data throughout the interval, and which is in harmony with the time dependent constraints of the model. This is likely to be especially important in the tropics, where if a balance between mass and motion exists, it is governed by the full equations.

But will this distinction make any real difference? The answer may depend to some extent on the usage of the resulting atmospheric representations. Hayden (1973) has suggested that there may not be a noticeable difference in operational use. An objective of 3.5-dimensional assimilation is the best representation of the atmosphere to serve as an initial state for an extended prediction. Applied to the same objective, four-dimensional assimilation may contribute only a slight refinement to that initial state, such that the prediction would begin with a better estimate of the initial tendencies. That slight refinement might prove to be of considerable importance in the tropics, and of occasional importance elsewhere. It must be remembered, however, that errors in the initial state are but one source of forecasting errors. It seems quite possible that small improvements in the initial state may be overwhelmed by model deficiencies in the subsequent prediction. On balance, the additional cost of fully four-dimensional assimilation probably cannot be justified. Operational data assimilation systems to be implemented in the next few years are therefore likely to be 3.5-dimensional.

However, a major objective of the Global Atmospheric Research Program is the production of global data sets for the diagnostic study of atmospheric behavior. For these purposes, it would be desirable to have available a nearly continuous representation of the atmosphere in order to examine more carefully the temporal evolution of important motion systems. Since such representations need not be generated on a daily basis, cost is a less important factor. This application may be most suitable for four-dimensional assimilation methods in the immediate future.

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